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Effect of divergence of light wave and alignment of crystal on the response of electrooptic modulators

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ABSTRACT

In this work we report on numerical investigations of the effect of the light beam divergence or imperfect crystal alignment on the response of electrooptic modulators. Resulting nonlinearities are discussed both in terms of nonlinear distorsions of modulators and as related to errors in measurements of quadratic electrooptic coefficients. Our calculations based on the Jones calculus have been performed for uniaxial crystals including KDP, and its isomorphs, and $LiNbO_3$. The results obtained confirm that either the response of the modulators or results of electrooptic measurements can be significantly affected by the light divergence or imperfections in the crystals alignment.

Keywords: Quadratic electrooptic effect, linear electrooptic effect, electrooptic modulators, KDP-type crystals, $LiNbO_3$, Jones matrices.

1. INTRODUCTION

Electrooptic crystals are widely employed in modulators. In recent years the electrooptic effect is also applied in high-voltage sensors and optical voltage transformers. In all these devices the knowledge of nonlinear distortions of the output signal is very important. Usually, the distortions are sensitive to the divergence of the light beam and crystal alignment. Measurements of the quadratic electrooptic coefficients can be affected by these factors as well (see, for example, Refs [1-4]).

The aim of this work is to calculate the nonlinear distortions and modulation efficiency in electrooptic modulators related to the light divergence. Our approach is based on the Jones calculus.[5-7] In addition, we consider experimental errors that can appear in measurements of the quadratic electrooptic coefficients.

2. METHOD

The intensity of the light passed trough the crystal and a quarter-wave plate sandwiched between crossed polarizer and analyzer is analyzed. In this work we take into account modulators with the light beam propagating along the optic axis of uniaxial crystal. The quarter-wave plate is introduced to provide the modulation at the middle point of the linear portion of the dependence of the relative light intensity on the induced phase difference, i.e. at the middle of the transmission characteristic of the modulator.

In our calculations the light entering the crystal plate and that emerging from the modulator is described by one-column Jones vectors[5,6] ε_o and ε

$$\varepsilon_o = \begin{bmatrix} E_{xo} \\ E_{yo} \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} E_x \\ E_y \end{bmatrix}, \tag{1}$$

where E_{xo}, E_x and E_{yo}, E_y are the x- and y-components of the electric field of the light-wave entering the crystal and emerging from the analizer, respectively. The light intensity can be found as

$$I = |E_x|^2 + |E_y|^2 . \tag{2}$$

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Optical elements, i.e. the quarter-wave plate, uniaxial crystal and analyzer, are represented by the Jones matrices $\mathbf{J}_1$, $\mathbf{J}_2$ and $\mathbf{J}_3$, respectively. The response of modulator ε is calculated from equation[5]

$$\varepsilon = \mathbf{J}_3\,\mathbf{J}_2\,\mathbf{J}_1\,\varepsilon_o\,. \tag{3}$$

Jones vectors cannot describe unpolarized light, therefore, in Eq. (3) the vector ε_o corresponds to the light passed through the polarizer. By omitting an expression connected with the phase of light-wave electric field, the vector ε_o is given as

$$\varepsilon_o = \begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix}, \tag{4}$$

where θ is the azimuth of polarizer.

General form of the Jones matrices for the objects under consideration can be written as[7]

$$\mathbf{J} = \begin{bmatrix} T_f\cos^2\alpha_f + T_s\sin^2\alpha_f\,e^{-i\Gamma} & \sin\alpha_f\cos\alpha_f\left(T_f - T_s\,e^{-i\Gamma}\right) \\ \sin\alpha_f\cos\alpha_f\left(T_f - T_s\,e^{-i\Gamma}\right) & T_f\sin^2\alpha_f + T_s\cos^2\alpha_f\,e^{-i\Gamma} \end{bmatrix}, \tag{5}$$

where T_f and T_s describe the transmission of fast and slow waves, respectively, α_f is the azimuth of the fast wave, and Γ is the phase difference between the slow and fast waves. In our calculations we assumed $T_f = 1$ and $T_s = 0$ for the polarizer and $T_f = T_s = 1$ for other plates. Γ in the quarter-wave plate can be dependent on the light divergence, however, we assumed the quarter-wave plate being thin enough to neglect this.

To investigate the effect of the beam divergence numerical calculations were performed. The deviations of the light propagation vector **k** from the optic axis in uniaxial crystals, i.e. from the [001] direction, we expressed in terms of the angles β and γ, as defined in Fig. 1.

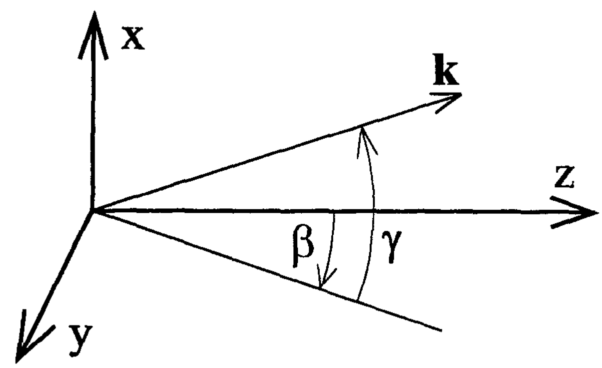

Fig. 1. Angles β and γ describing the deviation of the propagation vector **k** from the optic axis direction.

3. ELECTROOPTIC MODULATORS

Many important applications of the linear electrooptic effect involve the use of uniaxial crystals with the light path close to the optic axis. Previously, the effect of deviations of the laser beam from this direction has been considered in terms of the approach employing the optical index ellipsoid. It has been shown that the deviations can affect the depth of modulation at the fundamental frequency and give rise to non-linear distortions of the output signal (see, for example, Refs. [1-4, 8-14]).

According to the crystal symmetry and magnitudes of relevant electrooptic coefficients two different kinds of electrooptic modulators may be used. Devices with the electric field applied parallel and perpendicular to the direction of the light beam involve the use of the so-called longitudinal and transverse modulators, respectively. In this work we consider examples of these two types of modulators.

3.1. Longitudinal modulator - the example of the KDP crystal

Well known electrooptic uniaxial crystals that may be used in the longitudinal modulator are members of the potassium dihydrogen phosphate (KDP) family. This is because of relatively large value of the linear electrooptic coefficient r_{63}. It is readily shown that when both the light path and the modulating electric field are exactly parallel to the [001] direction, the relative modulation amplitude of the light intensity at the fundamental frequency I_ω / I_o is given by

$$I_\omega / I_o = \pi\, n_o^3\, r_{63}\, V_o / \lambda \tag{6}$$

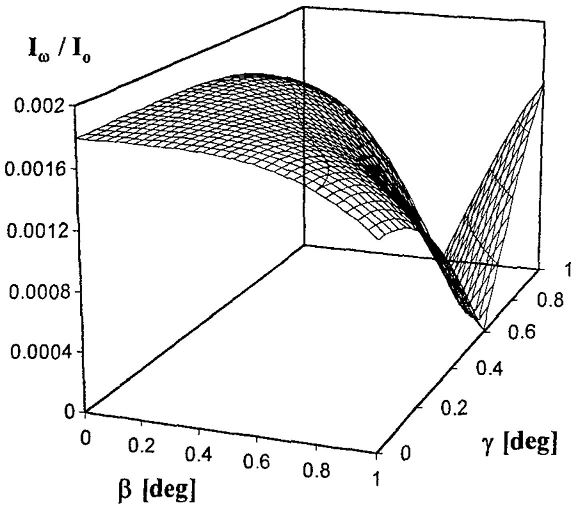

Fig. 2. Dependence of the relative modulation amplitude of the light intensity at the fundamental frequency I_ω / I_0 on the angles β and γ plotted for KDP.

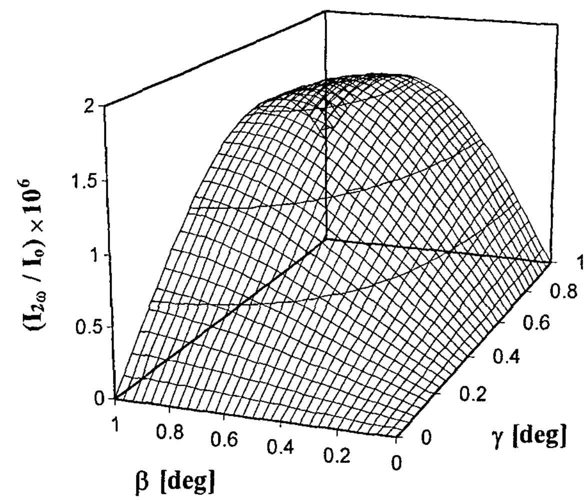

Fig. 3. Dependence of the relative modulation amplitude $I_{2\omega} / I_0$ at the second harmonic on the angles β and γ plotted for KDP.

Here, I_0 is the intensity of the light entering the crystal, n_0 is the ordinary refractive index, V_0 is the voltage amplitude of the modulating electric field and λ is the wavelength. When the light divergence is neglected the relative modulation amplitude of the light intensity at the second harmonic $I_{2\omega} / I_0$ is equal to zero and the modulation efficiency at the fundamental frequency is independent of the crystal length. The higher-order harmonics do not appear in the response of the modulator as well. Any deviation of the light from the [001] direction can lead to a decrease in I_ω and an increase in $I_{2\omega}$. Moreover, when the light divergence cannot be neglected, these factors become to be dependent on the crystal length. The changes in I_ω / I_0 and $I_{2\omega} / I_0$ obtained in this work by using the Jones calculus are illustrated in Figs 2 and 3. The relative nonlinear distortion described by the ratio $I_{2\omega} / I_\omega$ is presented in Fig. 4. Results shown in Figs 2-4 correspond to the crystal length 1 cm, the amplitude of modulating electric field 10^3 V/m, and the wavelength $\lambda = 630$ nm. The transmission axis of the polarizer is taken to be parallel to the [100] direction. The values of linear and quadratic electrooptic coefficients used in our calculations were taken from Refs 13-15.

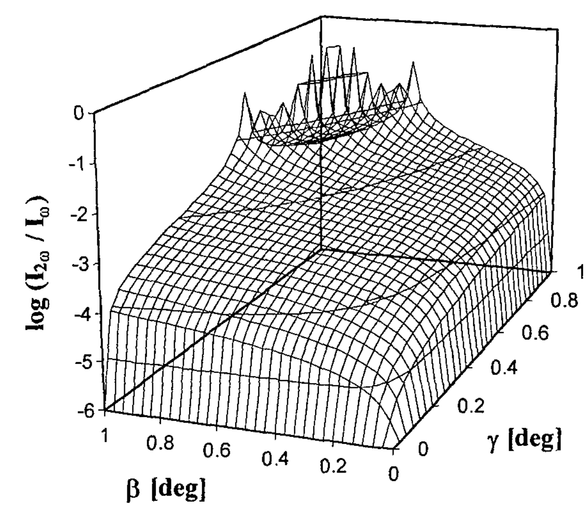

Fig. 4. Dependence of the relative nonlinear distortions $I_{2\omega} / I_\omega$ on the angles β and γ plotted for KDP.

3.2. Transverse modulator - the example of the $LiNbO_3$ crystal

The use of transverse modulators is often very convenient. When the thickness and length of the crystal are different, the increase of the ratio length to thickness leads to the decrease in the voltage necessary to drive the modulator. Moreover, relatively easily one can obtain an uniform electric field in electrooptic crystal.

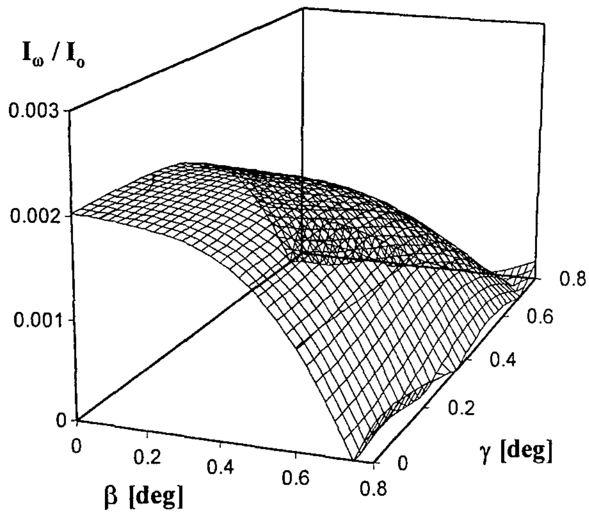

Fig. 5. Dependence of the relative modulation amplitude of the light intensity at the fundamental frequency I_ω / I_0 on the angles β and γ plotted for $LiNbO_3$.

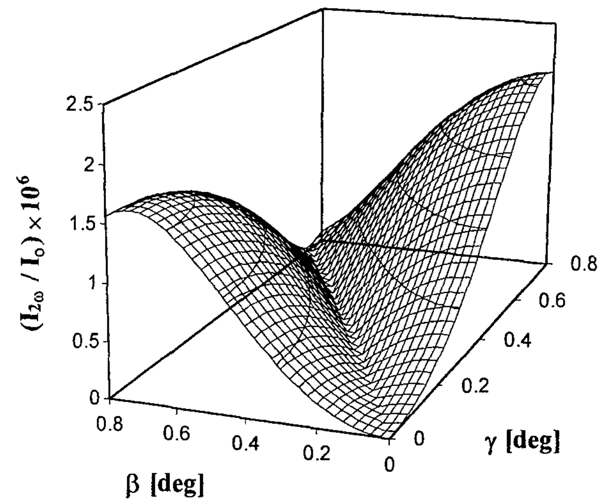

Fig. 6. Dependence of the relative modulation amplitude of the light intensity $I_{2\omega} / I_0$ at the second harmonic on the angles β and γ plotted for $LiNbO_3$.

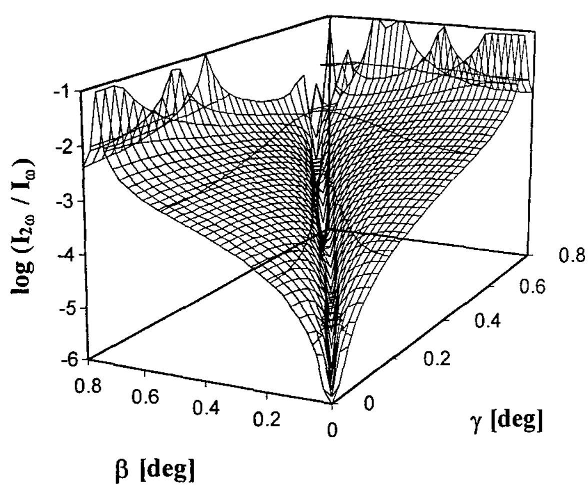

Fig. 7. Dependence of the relative nonlinear distortions $I_{2\omega} / I_\omega$ on the angles β and γ plotted for $LiNbO_3$.

The $LiNbO_3$ crystal is considered as the example of medium useful in applications in transverse modulators. It may be shown that when the modulating a.c. electric field is applied along the [010] direction and **k** is exactly parallel to the [001] direction, the relative modulation amplitude of the light intensity at the fundamental frequency I_ω / I_0 is given by

$$I_\omega / I_0 = \pi n_0^3 r_{12} V_0 L/\lambda t . \tag{7}$$

In Eq. (7) L is the light path in the crystal plate and t is the thickness of the crystal. Here, again, deviations of the light from the [001] direction can lead to the decrease in I_ω / I_0 and increase in $I_{2\omega}/I_0$. The changes in I_ω / I_0 and $I_{2\omega}/I_0$ obtained for the crystal length 1 cm, the amplitude of the modulationg electric field 10^3 V/m, and the wavelength $\lambda = 630$ nm are shown in Figs 5-7. The values of linear electrooptic coefficients used in our calculations were taken from Ref. 17. The angle between the transmission axis of the polarizer and the [100] direction was set at π/4.

4. MEASUREMENTS OF THE QUADRATIC ELECTROOPTIC EFFECT

Studies of electrooptic properties of crystals are of interest from the point of view both applications and an understanding of the nature of relevant nonlinear susceptibilities. The latter is important because the same nonlinearities are responsible for other nonlinear optical effects (see, for example, Refs 9,12-14,18-23). Furthermore, measurements of quadratic electrooptic coefficients allow to investigate the paraelectric-ferroelectric phase transition in ferroelectric crystals.[16]

As an example, in this work we present results obtained for the simulation of measurements of the quadratic electrooptic coefficient $g_{11} - g_{12}$. This coefficient may be determined in the KDP crystal with the electric field applied along the [100] direction and the laser beam passed in the [001] direction. Assuming the light divergence to be negligible, the $g_{11} - g_{12}$ coefficient may be experimentally determined as

$$g_{11} - g_{12} = \frac{4\lambda t^2}{\pi L n_o^3 V_o^2} \frac{I_{2\omega}}{I_o} \qquad (8)$$

Even small deviations of the light from the optic axis direction can significantly affect the measurement.[8,9] When allowing for the divergence, an erroneous value, denoted here as $g'_{11} - g'_{12}$ can be predicted from the response of the modulator. Results shown in Fig. 8 confirm earlier predictions drown from the analysis of the electric field induced changes in the optical indicatrix. Our plot describes the response of the crystal plate of length 1 cm, the amplitude of modulating electric field 10^5 V/m, and the wavelength $\lambda = 630$ nm. The values of linear and quadratic electrooptic coefficients used in our calculations were taken from Refs. [13-15]. The angle between the transmission axis of the polarizer and the [100] direction was set at $\pi/4$.

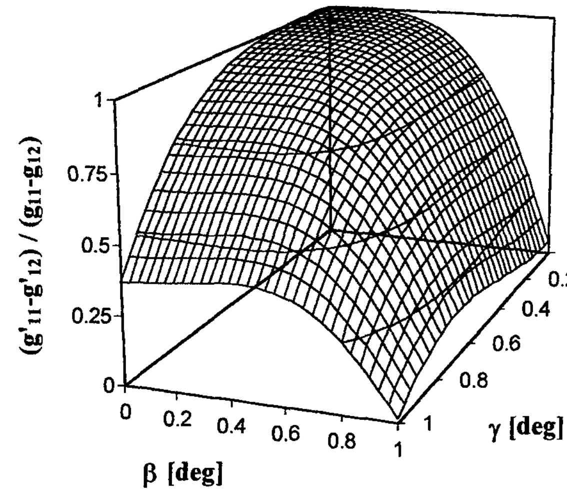

Fig. 8. The relative error in determination $g_{11} - g_{12}$ of KDP plotted against the angles β and γ.

5. CONCLUSIONS

Our results obtained within the framework of the Jones matrices approach support the observations related to the correlation between the efficiency of modulation at the fundamental frequency or non-linear distortions in the response of electrooptic modulators and the divergence of the light beam from the optic axis direction[1-4,8]. These earlier findings have been obtained employing the optical indicatrix approach. Our numerical calculations confirm previous predictions that in measurements of quadratic electrooptic coefficients the error depends strongly on the divergence of the light. The error may origin from two different sources. Relatively well recognised is the shift from the middle point of the linear part of the transmission characteristic of the modulator. The second one can result from a superposition of two linear in the field effects, namely, changes in the components of the impermeability tensor and the electric field induced changes in the azimuth of the fast wave.

REFERENCES

1. M.S. Ahmed and J.M. Ley, "Defects in the performance of electrooptic modulators using class $\bar{4}2m$ crystals", *J. Phys. D: Appl. Phys.* **4**, pp. 1637-1641, 1974
2. T. Tudor, "Harmonic structure of light modulated by longitudinal electrooptic effect in crystals of class $\bar{4}2m$ ", *J. Optics* **14**, pp. 161-168, 1983
3. P. Górski and W. Kucharczyk, "On the measurement of the quadratic electrooptic effect", *Phys. Stat. Sol. (a)* **100**, pp. K73-76, 1987
4. P. Górski and W. Kucharczyk, "On the measurement of electrooptic coefficients by the static polarimetric technique", *Optik* **83**, pp. 7-10, 1989
5. R.C. Jones, "A new calculus for the treatment of optical systems. I. Description and discussion of the calculus", *J. Opt. Soc. Am.* **31**, pp. 488-493, 1941
6. R.C. Jones, "New calculus for the treatment of optical systems. VIII. Electromagnetic theory", *J. Opt. Soc. Am.* **46**, pp. 126-131, 1956
7. I. Ścierski and F. Ratajczyk, "The Jones matrix of the real dichroic elliptic object", *Optik* **68**, pp. 121-125, 1984
8. P. Górski, W. Kucharczyk, "On the application of uniaxial crystals in modulators", *J. Optics* **20**, pp. 187-192, 1989
9. W. Kucharczyk, "Quadratic electro-optic effect and second-order strain derivatives of electronic susceptibility", *Physica B* **176**, pp. 189-208, 1992
10. M. Izdebski, W. Kucharczyk and R.E. Raab, "Effect of beam divergence from the optic axis in an electrooptic experiment to measure an induced Jones birefringence", to be published

11. M.J. Gunning, R. Ledzion, P. Górski and W. Kucharczyk, "Studies of the quadratic electrooptic effect in KDP-type crystals", International Conf. on Solid State Crystals '98: Single Crystal Growth, Characterization, and Applications, A. Majchrowski; J. Zielinski; Eds. *Proc. SPIE* **3724**, pp. 249-255, 1999
12. W. Kucharczyk, M.J. Gunning, R.E. Raab and C. Graham, "Interferometric investigation of the quadratic electro-optic effect in KDP", *Physica B* **212**, pp. 5-9, 1995
13. M.J. Gunning, R.E. Raab, P. Górski and W. Kucharczyk, "The quadratic electrooptic effect and estimation of antipolarization in ADP", *Ferroelectric Lett.* **24**, pp. 63-68, 1999
14. M.J. Gunning, R.E. Raab and W. Kucharczyk, "Magnitude and nature of the quadratic electro-optic effect in KDP and ADP single crystals", to be published
15. Landolt-Börnstein: *Numerical Data and Functional Relationships in Science and Technology*, New Series, Group III vol. 18, Springer Verlag, Berlin-Heidelberg-New York, 1984
16. P. Górski, R. Ledzion and W. Kucharczyk, "Application of quadratic electrooptic effect to investigate paraelectric-ferroelectric phase transition in KDP-type crystals", *Proceedinds of the Fifth International Conference on Intermolecular Interactions in Matter (IIM-5)* Lublin 1999, Politechnika Lubelska, Eds. K. Sangwal, E. Jartych and W. Polak
17. T.A. Maldonaldo and T.K. Gaylord, "Electrooptic effect calculations: simplified procedure for arbitrary cases", *App. Optics* **27**, pp. 5051-5066, 1988
18. W. Kucharczyk, "A bond-charge calculation of the quadratic electro-optic effect in LiF", J. Phys. C: Solid State Phys. **20**, pp. 1875-1880, 1987
19. W. Kucharczyk, "The first- and second-order strain derivatives of electronic dielectric constants in alkali halides", J. Phys. Chem. Solids **55**, pp. 237-242, 1994
20. W. Kucharczyk, "Bond polarizability approach to nonlinear phenomena in crystals", International Conf. on Solid State Crystals '98: Single Crystal Growth, Characterization, and Applications, A. Majchrowski; J. Zielinski; Eds. *Proc. SPIE* **3724**, pp. 69-74, 1999
21. W. Kucharczyk and F.L. Castillo-Alvarado, "Calculations of hyper-Raman tensor components of alkali halides", J. Phys.: Condens. Matter **11**, pp. 305-309, 1999
22. W. Kucharczyk, F.L. Castillo-Alvarado, P. Górski and R. Ledzion, "Calculations of nonlinear refractive index of alkali halides", Opt. Quant. Electron. **32**, in print, 2000
23. W. Kucharczyk, F.L. Castillo-Alvarado, P. Górski and R. Ledzion, "The effective band charge in alkali halides – another manifestation of the crystals ionicity", J. Phys. Chem. Solids, accepted to print